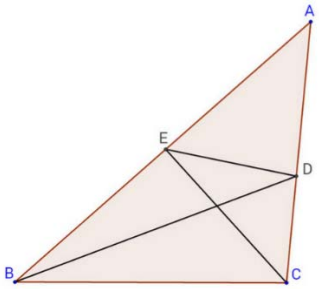
Math Club Worksheet: COMC Preparation #5

Warmup #1) A priest and bishop are in a church with three nuns. The priest tells the bishop that the age of all three nuns multiplies to 2450. The sum of the ages of all three nuns is twice the age of the priest." The bishop says he still doesn't know the age of the three nuns. So the priest gives him another clue "You are the oldest person in this group". How old is the bishop, the three nuns, and the priest? (Note: the priest and bishop knows each other's age)

Warmup #2)



In $\triangle ABC$, the bisectors of $\angle ABC$ and $\angle ACB$ meet AC and AB at D and E respectively, and $\angle ADE - \angle AED = 60^\circ$. Find the value of $\angle ACB$ in degrees.

Brilliant Level 4

Warmup #3)

Find the remainder when “a” is divided by 13. [brilliant Level 4]

$$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{23}=\frac{a}{23!}$$

Question #3) If “x”, “y” and “z’ are positive numbers that satisfy the system of equations above, find the value of $10000x+100y+z$. [Brilliant Level 5]

$$\begin{cases} x^2-yz=37 \\ y^2-xz=1 \\ z^2-xy=-35 \end{cases}$$

You are trapped in a small room with 4 walls. Each wall has a button that is either in an OFF/ON setting though you have no way of telling what the setting is.

When you press a button, you change its setting. If you can get all the buttons to have the same setting i.e. either all four are OFF or all four are ON, you are immediately set free.

In each Move, you can press either two buttons simultaneously or just one button. As soon as this occurs, if you haven't been set free, the whole room spins around you violently, leaving you completely disoriented so that you can never tell which side is which.

The starting position is completely at random (except not all four OFF or all four ON). Given any and every possible scenario, using optimal strategy, what is the least number of Moves needed to unquestionably guarantee escape from the room?

Let “ABC” be a right-angled triangle with the right angle at “C”. Let BCDE and ACFG be squares external to the triangle. Furthermore, let AE intersect BC at H, and let BC intersect AC and “K”. Find the size of $\angle DKH$: